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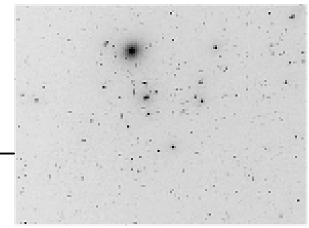


The Distance to the Hyades Star Cluster

Objective

To find the distance to the Hyades star cluster using the convergent point method; to recognize the uncertainties in using this method; to summarize the significant results obtained by the Hipparchos satellite and what these results mean for the distance scale of the Universe. [**Adapted from an exercise for Astronomy 200, University of Victoria, Victoria, BC, Canada.**]

Introduction

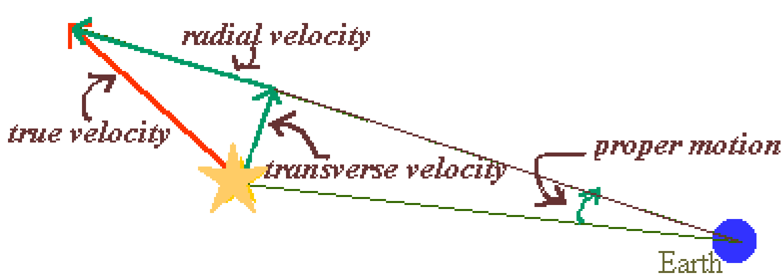
The nearest open cluster to us, the Hyades has been extremely important to us in our studies of the structure of the Galaxy, in how the Galaxy evolves, and in distances across the Galaxy. The determination of the distance to the Hyades indirectly sets the foundation for the entire cosmic distance scale through our understanding and calibration of Cepheid variables. It is a bit unsettling then to think that even over the past decade, distance determinations to this cluster have varied by as much as 15%. The problem? From ground-based telescopes, **reliable** parallaxes for individual stars can be measured out to about 40 parsecs. At 40-50 parsecs, the Hyades cluster is just beyond this distance. As a result, other (often ingenious) methods have had to be used.

Almost all distance determinations used for the Hyades have either been based upon the convergent point method or have been judged according to their agreement or disagreement with it. The geometry used is relatively simple, but before we discuss how to go about using the method, we must define and explain some important concepts.

First of all, stars are not really "fixed." They only seem that way to us because of our crude measuring instruments or our limited observational time period (less than 100 years). We can describe their space motion by measuring two quantities: the proper motion and the radial velocity.

A. **Proper Motion**: Proper motion (usually denoted by the Greek letter μ) is the angular change in position of a star across our line of sight, measured in arc seconds per year. For stars visible to the naked eye, the average proper motion is around 0.1 arcsec per year (remember how small an arc second is?). The largest proper motion award goes to Barnard's star: a whopping 10.27 arcsec per year. Proper motions are generally measured by taking photographs several years apart and measuring the displacement of the image of a star with respect to more distant background stars over that time period. Usually decades must elapse between successive photographs before a reliable measurement can be made. The proper motion of a star results from its "transverse velocity," or the component of its true space velocity perpendicular to our line of sight (see Fig. 1 at the right).

Figure 1



B. **Radial Velocity**: The absorption lines in a stellar spectrum will have their wavelengths shifted if that star has some component of its space velocity along our line of sight (see Fig. 1). This shift in wavelength (or frequency) is called the Doppler shift, and is analogous to what we hear happen to sound waves coming from a rapidly approaching and then receding siren. The radial velocity of a star is remarkably easy to measure: we just take a spectrogram of a star and measure the displacement, $\Delta\lambda$, of a spectral line from its expected position, λ. If **c** is the speed of light, the radial velocity, $\mathbf{v_r}$ is given by:

$$\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda}$$

Now, here is the problem. What we want is the transverse velocity of the star, $\mathbf{v_t}$. What we can measure is the proper motion and **radial** velocity of that star. Just as the transverse velocity of a train along a distant track cannot be discerned unless we know the distance to that train, so proper motion can be converted to transverse spatial velocity only if the distance to the star is known. Fortunately, the radial velocity of a star is already an absolute space velocity relative to the Earth -- it's the transverse velocity we must obtain via a distance determination. The trick is to figure out how to find the actual velocity without first knowing the distance!

For stars that are too distant for the measurement of a reliable parallax, a different geometrical method may be applied if those stars belong to a few close clusters. The fundamental principle involved is that if we know the actual velocity of a star across the celestial sphere, and if we can measure its proper motion (apparent angular velocity), then we can tell how far away the star is. In the same way, if you saw a car speeding along the horizon at a **known** velocity, you could tell how far away it was by observing how slowly it appeared to move. The trick is to figure out how to find the distance without first knowing the actual velocity!

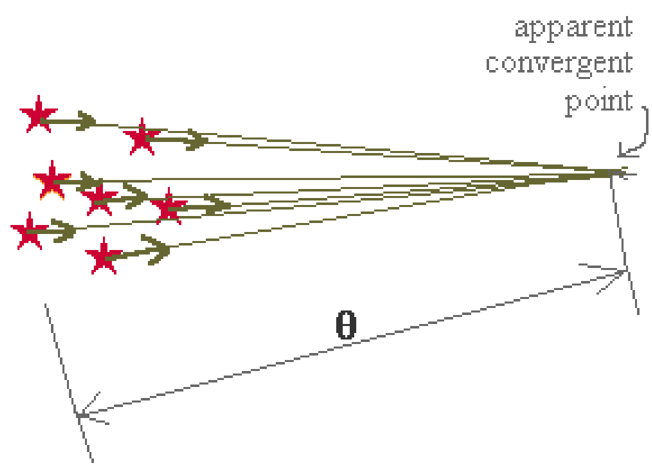


Figure 2

If we have a group of stars moving together, we may sketch the motion of each star in space as in Fig. 2. Just as two parallel railroad tracks appear to converge in the distance, so also will parallel star paths. This point of convergence is determined on a chart of the sky by simply extending the lines of proper motion of each star, and finding their point of intersection. The angle of sight between a star and the convergent point is measured on the chart, or in the sky. This angle is denoted by θ .

The important point that leads to our determining the distance to this cluster s that this angle, θ , is equal to the angle between the true space velocity of the star, **v**, and its radial velocity, $\mathbf{v_r}$ **(see Fig. 3)**. The radial velocity, $\mathbf{v_r}$, we can measure unambiguously. It is now a simple matter to solve the velocity right triangle for the velocity component tangent to the celestial sphere (seen as proper motion by the observer), $\mathbf{v_t}$. Review the geometry of this method, shown in Fig. 3 below, Make sure you understand the geometry and why this works.

$$v_t = v_r \tan\theta$$

We now have the actual transverse velocity, $\mathbf{v_t}$, the proper motion, μ, and can calculate the distance **D** (in parsecs) by using the equation:

$$\underset{\text{(pc)}}{D} = \frac{v_t \quad \text{(km/sec)}}{4.74\ \mu \quad \text{("/year)}}$$

[The derivation of this equation is given at the very end of this exercise.]

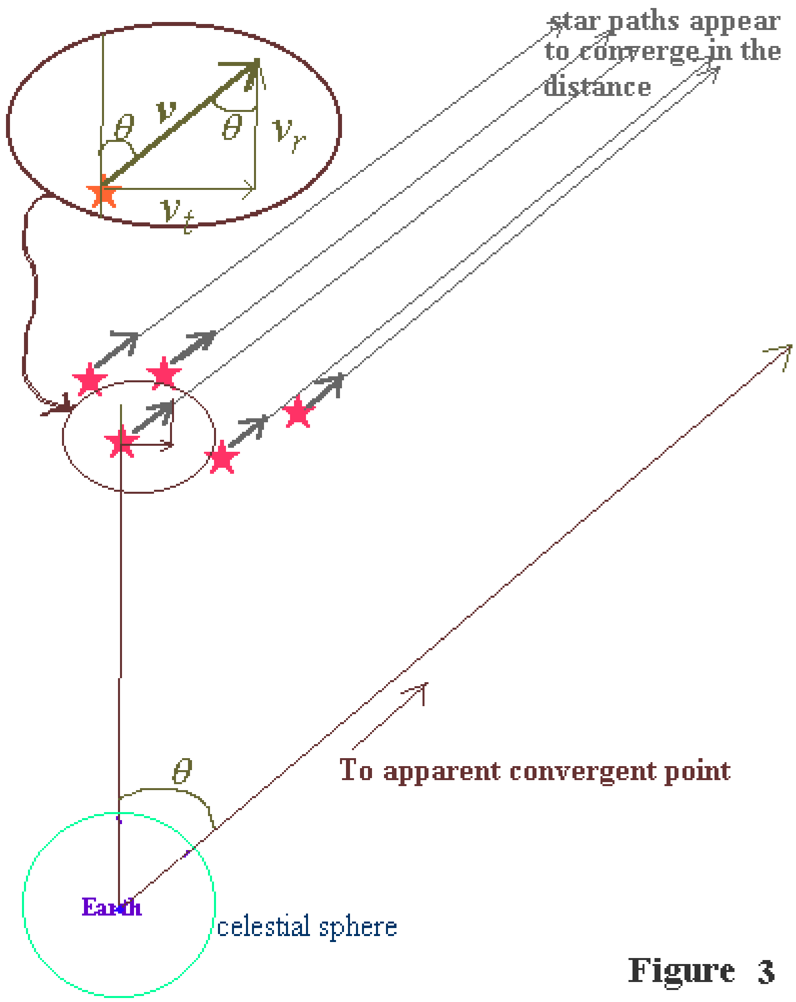


Figure 3

If this procedure is carried out for many stars in a cluster, an average of the distances calculated will be a good indication of the actual distance to the cluster. It is not expected that all stars in the cluster will all be at the same distance, as the cluster has some depth (see Fig. 5 at the end of this exercise). Also, even though the stars are moving together through space in the Galaxy, they also move about a common center of mass. The method also assumes that the cluster is neither expanding, contracting, nor rotating. The motion of the cluster must be large enough that accurate determination of cluster membership can be made.

Exercise

Table 1 lists the radial velocity and proper motion for each of the 10 stars indicated on this copy of the Hyades cluster (see Fig. 4 at the end of this exercise). Please read through the following instructions entirely before starting your work.

1. Print the copy of the Hyades cluster (Fig. 4) showing the member stars and vectors (arrows) depicting the direction and magnitude of the proper motion for those stars.
2. Tape another page to the left-hand side of this image, if needed. With a ruler, extend the x-axis scale onto the side sheet. (Note that this scale gives the right ascension of the stars in **degrees**. This will help you determine the angle θ easily.)
3. With a ruler, carefully "slice through" a group of vectors that seem to be pointing in nearly the same direction with straight lines (see sample below – **Don't do too many, or you will make your job more difficult!**) Because this cluster of stars is traveling through space as a unit, the lines will seem to converge. But, since within the cluster the stars are moving around the center of mass in addition to progressing along with the general cluster, **there will not be exact convergence**. In addition, there are errors in the determination of each proper motion, and as you can see,

some crowding of the data. These inaccuracies are great enough that we ignore the fact that we are working with a 3-dimensional cluster on a 2-dimensional page; in fact, to do this most accurately, these extensions should be slightly curved. (Thought question: Why curved?)

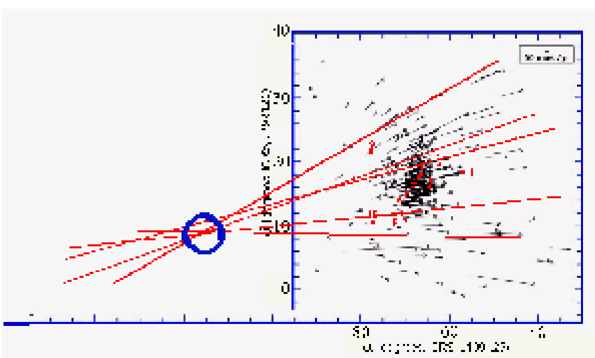
4. Decide where the density of the lines is greatest and circle that point. (You may wish to use a coin to draw your circle: if the convergent point is tight, use a dime; if it is hard to determine, use a quarter; if it is extremely imprecise, use a jar lid!) This circle gives you some indication of how precise your determination is.

5. Complete the data in Table 1 and find the distance to the Hyades cluster by averaging the distance to each of the 10 stars.

    a. Find the transverse velocity for each of the 10 stars marked in Fig. 4. To do this, measure the angle θ between each of the 10 stars and the convergent point, using the right ascension, marked in degrees of arc instead of hours of time, along the x-axis to do this. You may wish to devise your own measuring tool or determine a scale to use with a ruler. Then, use the given radial velocity and the appropriate equation given above.

    b. Find the distance to each star using the transverse velocity and the proper motion for each of the 10 stars, using the appropriate equation given above.

    c. Determine an approximate error in your measurement of the distance based upon the size of your "error" circle. That is, for your determination, what is the farthest and what is the nearest the cluster can be?



An example of what your cluster diagram may look like when finished with the above steps 1 – 5.

Questions

1. Give two reasons why distances to individual stars vary from each other.

2. Could the distance to the Hyades be determined by annual parallax? Discuss.

3. Using criteria of your choice, what is the diameter of the Hyades in parsecs? Be sure to summarize how you figured this out.

4. What is the mean distance in light years to the Hyades? What is the diameter of the Hyades in light years?

5. The Hipparcos satellite was able to measure the parallaxes of 200 Hyades stars very reliably. The distance to the Hyades's center of mass has been pinpointed at 46.34 +/- 0.27 parsecs, or 151 +/- 0.9 light years.
   a. Quantitatively compare your results to the Hipparcos results.

   b. In general, the convergent point method gives larger distances than other distance-finding methods used for clusters. Does the value you calculated follow this trend? Theorize on why the distances using this method tend to be systematically larger. (We are looking for **your** thoughts here.)

   c. Why is it significant that Hipparcos has parallaxes for so many stars?

   d. If the distance to the Hyades has been better determined by the Hipparcos satellite through measured parallaxes, is the convergent point method still useful? Think of a reason why the method is still .

6. The Hyades star cluster forms one of the fundamental, bottom steps of the distance scale for the entire universe. What implication does this more accurate (and precise) measurement of the distance to the Hyades have for the size of the Universe? (This is meant to be an easy question.)

Distance to the Hyades Cluster

Table 1: Data and Distance for the 10 Marked Stars

Star Number on Fig. 4	Radial Velocity (km/sec)	Angle θ	Transverse Velocity (km/sec)	Proper Motion (arcsec/year)	Distance (parsecs)
1	31.6			0.151	
2	31.0			0.121	
3	36.6			0.124	
4	38.3			0.114	
5	43.8			0.120	
6	38.2			0.098	
7	39.3			0.100	
8	38.6			0.108	
9	43.6			0.079	
10	38.8			0.065	
				Average Distance:	
				Error in Distance:	

The Hyades Star Cluster in 3-Dimensions

This 3-D image comes from the June 1999 issue of Sky & Telescope, and is based on the Hipparcos distance to each of the stars. To see the 3-D view from the computer screen, first scroll the page or move your head so that your eyes are level with the images. Stare at the middle of the space between the two images. Relax your eyes and gaze into the distance "through" the screen. The two images will seem to split and form an additional image in the middle. If you get the split images to line up exactly on top of each other, they will fuse into one center frame, revealing the 3-D effect.

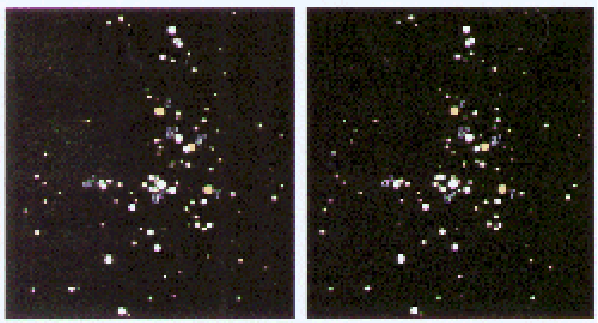
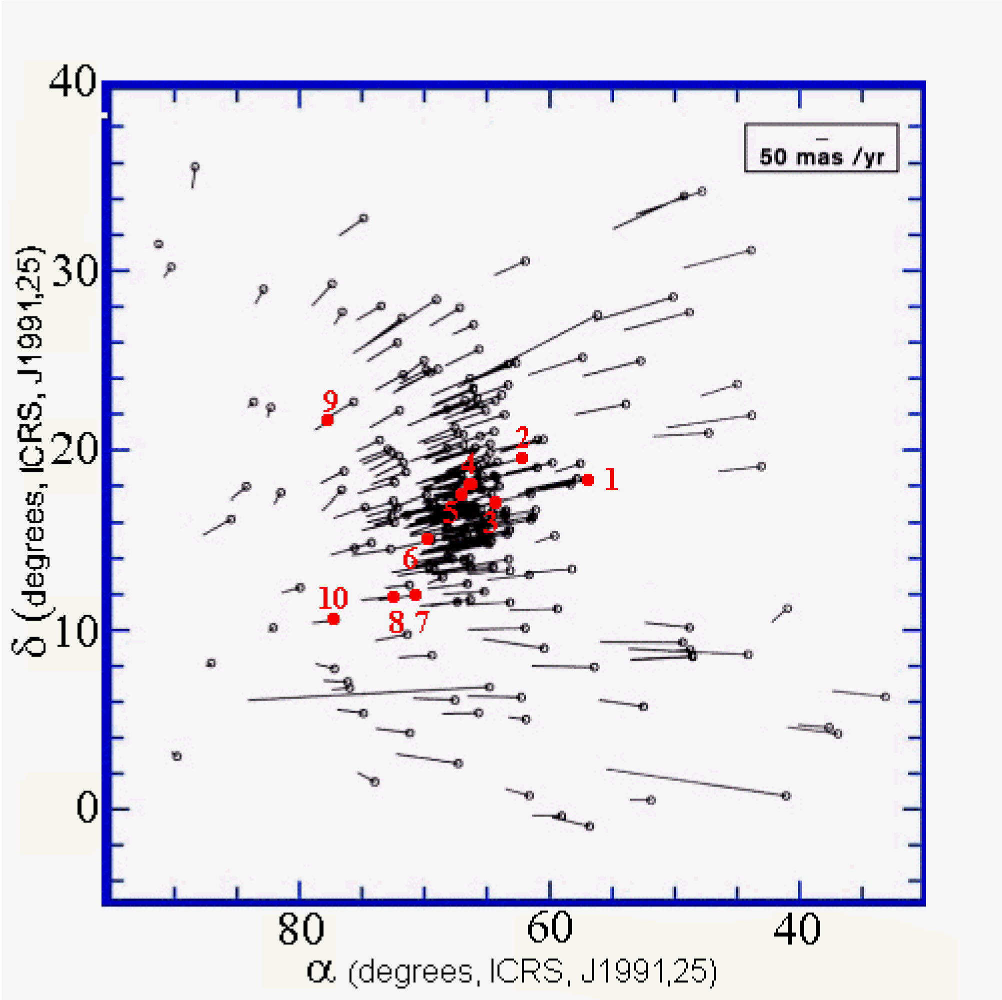
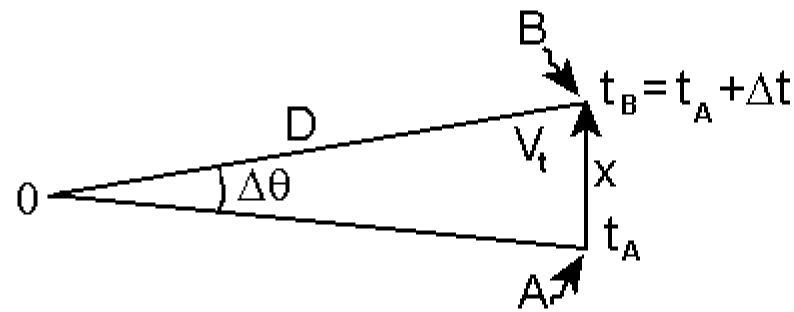


Figure 5

It is not recommended that you do this for any extended period of time. A color image is available on-line.


50 mas /yr
δ (degrees, ICRS, J1991,25)
α (degrees, ICRS, J1991,25)
40
30
20
10
0
80
60
40
1
2
3
4
5
6
8,7
9
10

Derivation of the distance equation used in the Hyades lab



Transverse motion of a star

Let's say that we observe a distant object, far away at a distance **D**, that shows a transverse velocity $\mathbf{V_t}$. At a given time, the observer notes the object at location **A** at a time $\mathbf{t_A}$. After a given amount of time, the observer notes the same object at location **B**. The object has traveled an actual distance of **X**, which at the distance **D** is measured as the small angle $\Delta\theta$. How can we determine the actual distance, **D**?

The time the object has taken to travel the distance **X** is Δt, and thus we use the relationship between distance traveled, velocity, and time: $x = v_t \Delta t$

Since the distance **D** is extremely large, we may use the small angle formula to obtain the relationship between **D** and **X**:

$$\frac{D}{206265\,\text{arcsec/radian}} = \frac{x}{\Delta\theta\,\text{arcsec}}$$

Note that since we **always** measure $\Delta\theta$ in arc seconds, we must do some converting between arc seconds and radians (there are approximately 206,265 arc seconds in a radian. Also, **D** and **X** must be measured in the same units.

If we substitute $x = v_t \Delta t$ for **X**, this equation becomes

$$\frac{D\,(\text{km})}{206265\,\text{arcsec/radian}} = \frac{v_t\,(\text{km/sec})\Delta t\,(\text{sec})}{\Delta\theta\ \text{arcsec}}$$

The proper motion of a star, μ is defined by $\Delta\theta/\Delta t = \mu$. To get the equations into the form where we are using the common units for proper motion--arc seconds per year--and distance--parsecs--we must do some unit conversions. In final form, the equation we use is

$$D(pc) = \frac{v_t\,(\text{km/sec})}{4.74\mu\ (\text{arcsec/yr})}$$

where the number **4.74** comes from

$$4.74 = \frac{3.09\times10^{13}\ (\text{km/pc})}{3.16\times10^{7}\ (\text{sec/yr})\ \times 206265\ (\text{arcsec/radian})}$$

Once again, the small-angle approximation makes our calculations much easier, although it may not look like it at first glance!